CENTER FOR TRANSPORTATION RESEARCH

An Adaptive Signal Control Method Involving the Lighthill Whitham Richards Model Using Mixed Integer Linear Programming

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An adaptive signal control framework for a single intersection is developed. Both the traffic volume prediction model (LP) and the signal optimization model (MILP) are derived from the Barron-Jensen/Frankowska (B-J/F) solution to the LWR model. Moreover, a approximated algorithm is proposed to solve the optimization model fast to make this method applicable.

Introduction

Kernel components of adaptive control algorithms:

- Traffic volume prediction: detectors;
- Signal optimization model: traffic flow models.

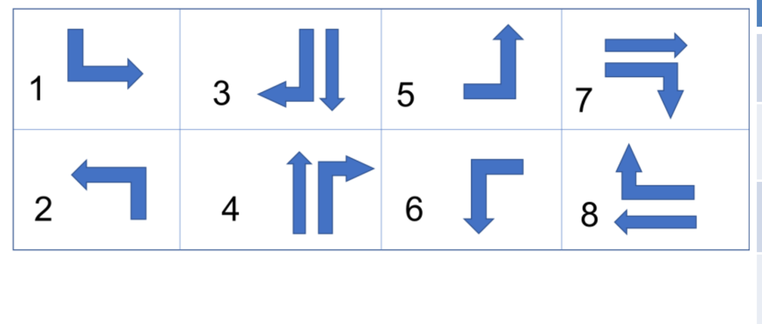
Methods proposed in the past decades:

- SCOOT (1) and SCATS (2): Efficiency decreases with rapidly varying traffic conditions;
- OPAC (3) and RHODES (4): Rolling horizon scheme is utilized to address limitation of the distance between the intersection and upstream detectors;
- Zheng and Recker (5) : Assumes Poisson arrivals and constant rate in each iteration.

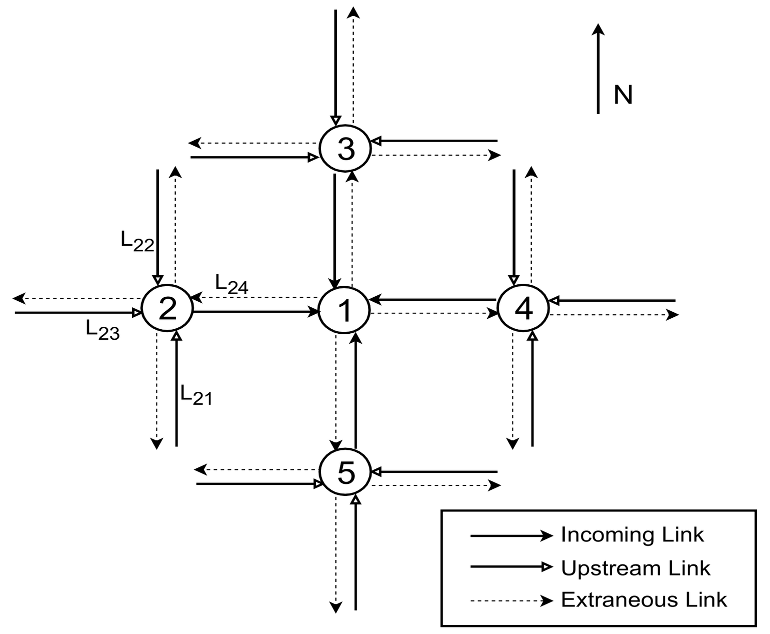
Contributions of this paper:

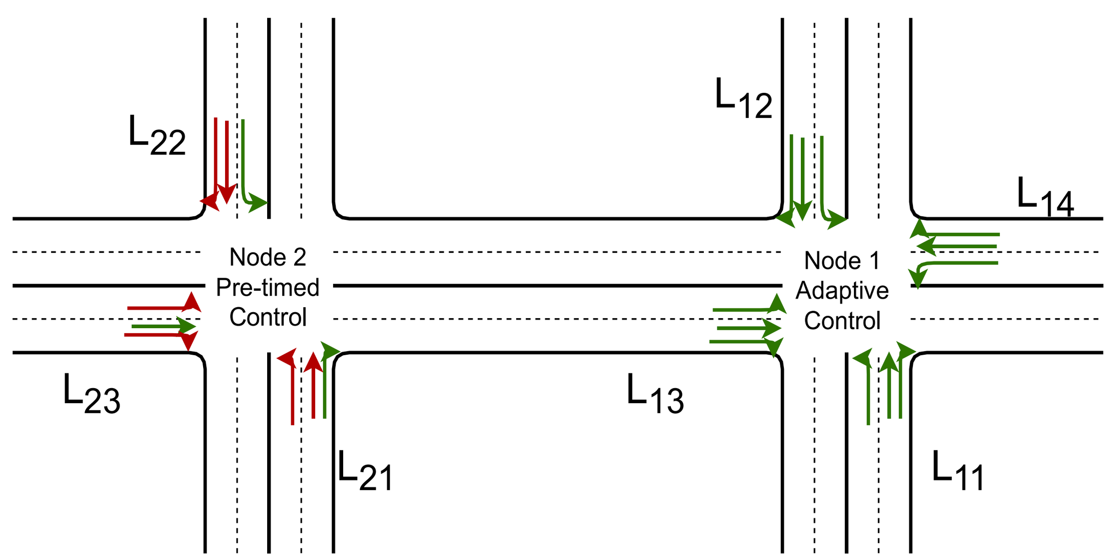
- ➢ Propose a traffic volume prediction model resorted to the current traffic state in a surveillance system;
- ➢ Propose a signal optimization model based on the Barron-Jensen/Frankowska (B-J/F) solution to the Lighthill-Whitham-Richards (LWR) model;
- ➢ Demonstration through microscopic simulation

Network Layout



Phase number	1	2	3	4
Movements (nodes 3 and 5)	1 & 2	3 & 4	5 & 6	7 & 8
Green Time (s)	12	28	12	28
Movements (nodes 2 and 4)	7 & 8	5 & 6	3 & 4	1 & 2
Green Time (s)	28	12	28	12





Model Description

Prediction:

Predict the outflow from upstream links:

$$\max_{q_{out}^{j}(t,i)} \sum_{j\in J}\sum_{i\in I}\sum_{t\in N} q_{out}^{j}(t,i)(n_{\max}-t)$$

s.t. *Compatibility* $\quad \forall i\in I, j\in J$

$$q_{out}^{j}(t,i)\le c(i)p^{j}(t,r) \qquad \forall i\in I_r, t\in N, j\in J, r\in R$$

$$q_{out}^{j}(t,i)\ge 0 \qquad \forall i\in I, t\in N, j\in J$$

Predict the inflow of incoming links:

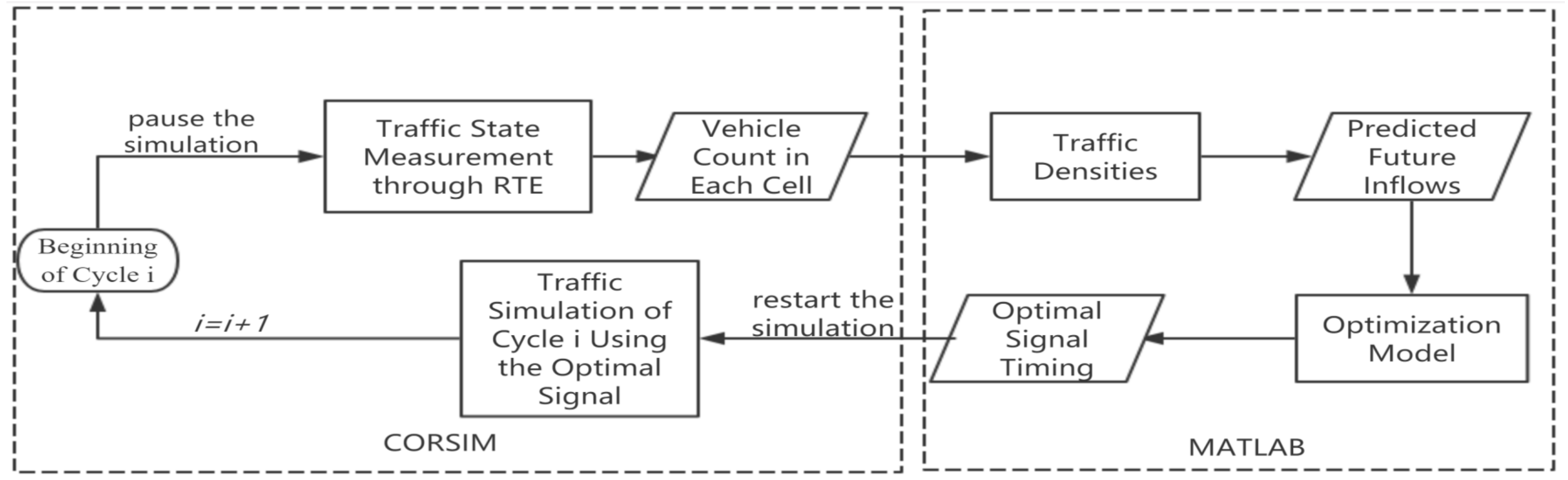
$$\begin{cases} q_{in}(t,5)=\left(q_{out}^{2}{}^{*}(t,1)\beta^{2}(1)+q_{out}^{2}{}^{*}(t,4)\beta^{2}(4)+q_{out}^{2}{}^{*}(t,7)\beta^{2}(7)\right)\alpha_5 \\ q_{in}(t,7)=\left(q_{out}^{2}{}^{*}(t,1)\beta^{2}(1)+q_{out}^{2}{}^{*}(t,4)\beta^{2}(4)+q_{out}^{2}{}^{*}(t,7)\beta^{2}(7)\right)\alpha_7 \end{cases}$$

Optimization:

Signal constraints:

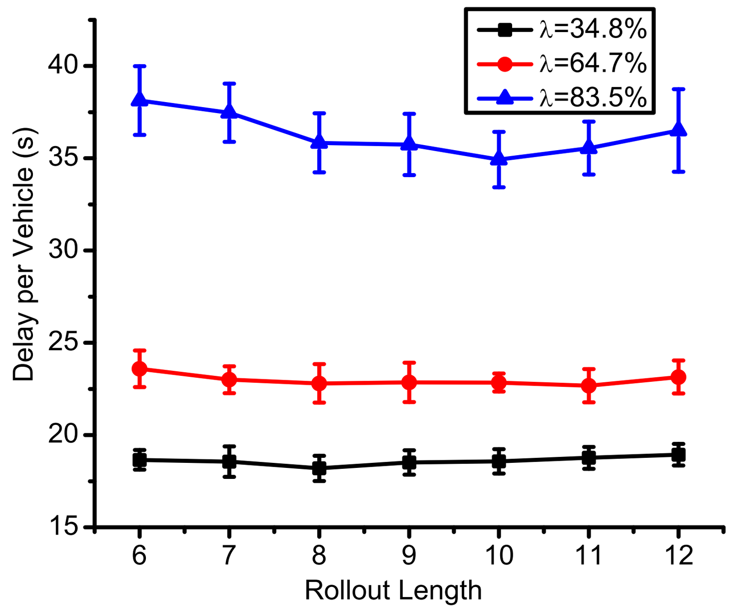
$$\begin{cases} \sum_{r\in[1,r_{\max}]} p(r,t)\le 1, & " t\in[1,n_{\max}] & (a) \\ \sum_{t\in[1,n_{\max}]} p(r,t)\ge 1, & " r\in[1,r_{\max}] & (b) \\ p(r,t+1)+\sum_{\substack{r'\in[1,r_{\max}] \\ r'\ne r}} p(r',t)\le 1, & " r\in[1,r_{\max}], " t\in[1,n_{\max}-1] & (c) \\ \sum_{r\in[1,r_{\max}]}\sum_{i=t}^{t+g_{\min}} p(r,i)\ge g_{\min}, & " t\in[1,n_{\max}-g_{\min}] & (d) \\ \sum_{r\in[1,r_{\max}]} p(r,1)\ge 1, & & (e) \\ \sum_{r\in[1,r_{\max}]} p(r,n_{\max})\le 0, & & (f) \\ q_{out}(i,t)\le c(i)p(r,t), & " i\in I_r, " r\in[1,r_{\max}] & (g) \end{cases}$$

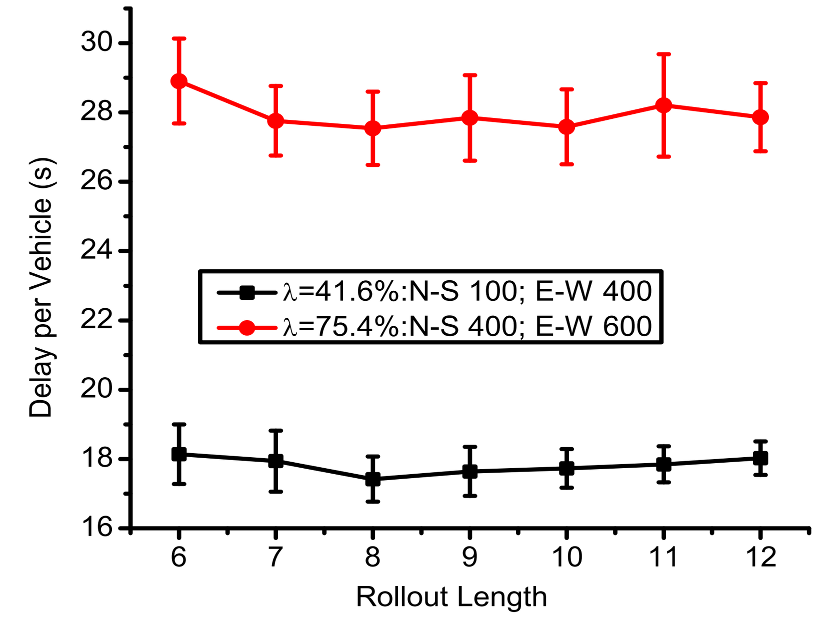
Case Study



Balanced	200			400			600		
	Entry flow (vph)								
	Exact	Rollout	HCM	Exact	Rollout	HCM	Exact	Rollout	HCM
Mean	17.6	22.8	44.5	22.5	22.8	28.3	33.9	34.9	44.5
Reduced (%)	**23.6**	**19.4**	-	**20.6**	**19.4**	-	**23.8**	**21.6**	-
p-value	**2.2e-7**	**2.0e-7**	-	**6.0e-7**	**2.0e-7**	-	**2.4e-7**	**8.2e-7**	-
λ (%)	40.2			65.8			83.5		
Unbalanced	N-S 100 E-W 400			N-S 200 E-W 500			N-S 400 E-W 600		
	Exact	Rollout	HCM	Exact	Rollout	HCM	Exact	Rollout	HCM
Mean	17.5	17.7	24.4	20.8	21.4	26.2	27.2	27.6	33.1
Reduced (%)	**28.3**	**27.2**	-	**20.7**	**18.1**	-	**17.8**	**16.6**	-
p-value	**6.8e-11**	**2.7e-10**	-	**2.1e-8**	**1.7e-7**	-	**1.2e-6**	**2.0e-6**	-
λ (%)	41.6			57.2			75.4		

Effect of Rollout Length:





Future Research Direction

- ➢ Consider the uncertainty in the mean of volume if only historical data is available
- ➢ Take start-up delay into account;
- ➢ Develop a model for arterial street and network level.